The teaching of linear programming in different disciplines and in different countries

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The Teaching of Linear Programming in Different Disciplines and in Different Countries

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Abstract: This paper discusses an online survey of linear programming (LP) lecturers in four countries in various disciplines. The study uses Biglan's [1, 2] classification of disciplines to show that courses in hard-pure and hard-applied subjects were more likely to teach theoretical aspects of linear programming whilst the hard-applied and soft-applied subjects looked more at the application. Further, the soft-applied disciplines were more likely to utilize software during the teaching of the topic. Also, US lecturers were more likely to teach theoretical aspects of LP whilst the UK lecturers were more likely to use common software such as spreadsheets rather than dedicated LP or maths software.

1. Introduction

Linear programming (LP) is used to solve a set of linear inequalities to achieve an optimised solution. It is taught mainly at the tertiary level and may form part of the undergraduate and postgraduate curriculum. LP is taught in various disciplines particularly in mathematics, engineering and business. It is often presented in courses such as management science or operations research (MS/OR). Despite the presence of LP in this range of disciplines, there has been limited research into how LP as a stand-alone topic is taught particularly in disciplines other than business.

This research sought to investigate how LP was taught in various disciplines and the types of software employed. The research focused on the three main topics of LP: the formulation of the problem; the methods used for solving the problem particularly with respect to software; and what kind of sensitivity analysis was conducted. Further investigation looked at how the teaching may differ in four English-speaking countries since previous LP studies have mainly focused on the USA [3-5].

This research is influenced by a study by Albritton *et al.* [6] in which they investigated the content of MS/OR courses in MBA degrees. Their research covered a number of topics including statistics, LP and simulation. As such they were not able to provide an in-depth analysis of how each of these topics was taught. This paper is concerned with only one of these topics, LP, as it is one of the few topics that is taught in a variety of disciplines and can allow a comparison of disciplinary teaching from a common basis.

2. Research Design

The research used an online survey questionnaire. An online LP questionnaire was sent via an email link to 311 lecturers in four countries: Australia, New Zealand, USA and the UK. The email addresses of the lecturers were obtained by searching the internet via Google® for the key phrase 'linear programming' in websites with an .edu or .ac URL. The questionnaire had four sections.

The first section dealt with basic information pertaining to the LP module, in particular the type of software employed, the discipline in which the course resides, the disciplines the students are from and the level of the course such as undergraduate or postgraduate. The next three sections dealt separately with the teaching of the LP topics of formulation, solution and sensitivity analysis of LP. The concepts of 'coverage' and 'intensity of coverage' from Albritton *et al.* [6] were employed. Coverage refers to whether a topic such as formulation was taught in the course whilst degree of coverage indicated to what extent a topic was taught. In this study, the intensity of coverage was measured using a 5 point scale. The scale was coded as 1 = 'no coverage' to 4 = 'extensive coverage'. The fifth point was named as 'not sure' to minimize 'fence sitting' by the respondent. Coverage was coded as no coverage (1 = no coverage) and coverage (2, 3 or 4 = coverage). Figure 1 illustrates this type of question.

Figure 1: Example of a coverage question

When teaching linear programming, to what extent do you cover problem/ model formulation?
( ) No coverage
( ) Slight coverage
( ) Some coverage
( ) Extensive coverage
( ) Not sure

A second type of question and scale was devised for these three topics to measure the extent that software was incorporated in the teaching of these topics. These were referred to as delivery questions. Again a 5 point scale was used with -2 = 'predominantly whiteboard/ slides/handouts' to 2 = 'predominantly computer demonstrations'. The zero point thus meant there was equal use of computer demonstrations and whiteboard/slides/handouts. The fifth point again was labeled as 'not sure' to avoid fence sitting (see Figure 2).

Figure 2: Example of a method of delivery question

How do you deliver the problem/ model formulation section?
( ) Predominantly whiteboard/ slides/handouts
( ) Mostly whiteboard/ slides/handouts with some computer demonstrations
( ) Mostly computer demonstrations with some whiteboard/ slides/handouts
( ) Predominantly computer demonstrations
( ) Not sure

For the section on LP solution, delivery questions were also asked in reference to graphical solutions, simplex algorithm, revised simplex algorithm and the interior point method. There was an additional point added to the scale, 6 = 'not taught'. From these delivery questions, coverage was also determined for these solutions. Similar delivery questions were asked with respect to the graphical and computer sensitivity analysis in the sensitivity analysis section.

3. Results

The response rate for this survey was 25% with the lowest response rates from Australia and New Zealand. This response rate is similar to that received by Albritton *et al.* [6] and Jordan *et al.* [7] in their surveys of MS/OR programmes.

3.1 Discipline Analysis

The survey collected data from courses in various disciplines such as agriculture, business studies, computer science, mathematics and engineering. The disciplines were

grouped using Biglan's classification [1, 2]. The disciplines were grouped as 'hard' or 'soft' and 'pure' or 'applied'. For example, a course in engineering is classified as hard-applied as it is rooted in a clearly delineated paradigm and has an application nature [1, 2]. Only two other classifications were used: hard-pure (e.g. mathematics) and soft-applied (e.g. business studies) as no soft-pure (e.g. history) courses were found.

The three LP topics had almost the same percentage coverage with formulation having the highest overall coverage (100%) followed by the solution (96%) and the sensitivity analysis (86%). Although formulation was covered in all the courses, the mean intensity of coverage tended to be higher, although not significantly in the soft-applied disciplines (3.5) than in the hard-pure (3.2) and hard-applied (3.2) disciplines. Whilst a solution method was found to be taught in all courses in the hard-pure and hard-applied, only 77 % of the soft-applied courses taught a solution method (see Table 1). These results are quite similar to those of Albritton *et al.* [6].

Table 1: Comparison of coverage of linear programming sub-topics for the disciplines and Albritton *et al.* [6] study

LP Topic	Hard-Pure	Hard-Applied	Soft-Applied	All	Albritton *et al.*
Formulation	100%	100%	100%	100%	91%
Solution Methods	100%	100%	77%	96%	-
Graphical Method	91%	93%	69%	88%	79%
Simplex Method	86%	76%	23%	71%	19%
Revised Simplex Method	52%	59%	0%	45%	-
Interior Point Method	9%	24%	0%	13%	-
Sensitivity Analysis	77%	90%	100%	86%	83%
Graphical	69%	79%	77%	74%	-
Computer Printout/ Output	58%	83%	85%	72%	-

- : Albritton *et al.* did not cover this sub-topic

The solution methods that the soft-applied disciplines taught were mainly the graphical method (69%) with some simplex algorithm (23%), which appeared to be the two most popular solutions in all the disciplines. Whilst interior point method and revised simplex method were taught in both the hard-pure and hard-applied disciplines, in general, they were not taught in most courses.

For the sensitivity analysis, the soft-applied (100%) and hard-applied disciplines (90%) had a higher coverage than the hard-pure disciplines (77%) although this was not statistically significant. However, the intensity of coverage for sensitivity analysis was significantly higher in soft-applied (3.4) and hard-applied (3.1) than in the hard-pure (2.6) disciplines. Meanwhile, whilst 81% of hard-applied courses taught both the graphical and computer sensitivity analysis, only about 59% of the hard-pure and 62% of the soft-applied courses covered both.

With respect to software usage, about 84% of the courses employed one or more software packages. However, only about half used software for computer demonstrations during the teaching of formulation, the sensitivity analysis or teaching one of the solution methods. The remaining courses perhaps used the software for computing solutions. The most popular software in use were spreadsheets (48%) and LP dedicated software (29%). Whilst spreadsheets were popular in the soft-applied and hard-applied courses, LP dedicated software were used almost equally in all the disciplines. The hard-pure disciplines seem to favour mathematical software (e.g. Maple) and LP dedicated

software (e.g. Lindo). These results seem to corroborate the findings from Albritton *et al.* [6] for the soft-applied disciplines such as business.

Table 2: Type of software used across the disciplines and in *Albritton et al.* [6] study

Type of Software	Hard-Pure	Hard-Applied	Soft-Applied	All	Albritton *et al.*
Spreadsheet	20%	69%	77%	48%	76%
LP software	23%	35%	31%	29%	-
Spreadsheets and/ or LP	37%	72%	92%	60%	88%
Maths software	23%	24%	0%	20%	-
Math and/ or LP	46%	55%	31%	47%	34%
Other	14%	7%	8%	10%	-
No Software	26%	10%	8%	17%	-
Any Software	74%	90%	92%	83%	-

- : Albritton *et al*. did not cover this software

From the delivery questions, a computer demonstration score was developed for each discipline. It was found that most disciplines did not rely heavily on computer demonstrations for teaching. However, soft-applied disciplines (with a score of -0.7) used significantly more computer demonstrations *(*$F(2,74) = 8.674$, $p<0.001$*)* than the hard-applied (score -1.2) and hard-pure (score -1.6) disciplines.

3.2 Country Analysis

There are similarities in the coverage of LP formulation between the countries, however, there are differences in the coverage of the solution methods (see Table 3).

Table 3: Percentage of courses covering the linear programming sub-topics in the four countries

Linear Programming Topics*	Australia	N. Zealand	UK	USA	Total
Formulation	100%	100%	100%	100%	100%
Solution	100%	82%	97%	100%	96%
Graphical Method	83%	82%	88%	95%	88%
Simplex Method	83%	36%	76%	75%	71%
Revised Simplex Method	45%	36%	31%	74%	45%
Interior Point Method	8%	0%	0%	45%	13%
Sensitivity Analysis	83%	91%	85%	85%	86%
Graphical	83%	64%	74%	75%	74%
Computer Output/ Printout	73%	91%	61%	80%	72%

* Percentages do not include 'not sure' and missing values

Whilst the graphical method was taught almost equally in most countries, in New Zealand the odds of a lecturer teaching the simplex algorithm was half that of all the other countries ($\chi^2(3) = 8.009$, $p=0.046$*)*. These results possibly are not due to the number of courses in particular disciplines since the statistics from the study suggests that there is a similar distribution of courses in the various countries ($\chi^2(6) = 5.998$, $p=0.423$*)*, and it is therefore appropriate to discuss variations across disciplines separate from variations across countries. Further, the more intense mathematical solutions such as the revised simplex method and the interior point methods appeared to have at least a third more coverage in USA than in any other country. Also, most of the courses in all the countries seem to use at least one software package when teaching LP, however, the UK lecturers were almost half as likely to use dedicated LP and/ or mathematics programming software (e.g. Maple) than the other countries ($\chi^2(3) = 8.092$, $p=0.044$*)*.

4. Discussion and Conclusions

Although, the survey did have some limitations in having low response rate perhaps as a result of an online survey, the data was still able to extend previous research into the teaching of LP by investigating across various disciplines and countries. It found that the classification by Biglan seems to hold true in the teaching of LP. In particular, the applied courses (both hard and soft) were more concerned with the practical applications such as the interpretation of the sensitivity analysis than the pure courses. Further, the hard courses (both pure and applied) were the ones that were more theoretical in their perspectives and hence taught the more intensive mathematical solutions such as the revised simplex method and interior point method. The lecturers in the soft-applied disciplines however did not seem keen to teach solution methods excepting the popular solution methods such as the graphical and simplex method which even so were covered more by the hard-pure and hard-applied disciplines.

The hard-applied courses appear to act as a bridge between the teaching in the hard-pure and soft-applied disciplines. For example, the hard-applied courses had similar coverage in topics between itself and the hard-pure and soft-applied courses, but there were less coverage commonality between the two latter disciplines. This is seen more clearly in the software usage, where the hard-applied subjects used both spreadsheets and LP dedicated software almost as frequently as the soft-applied but yet it also used maths software almost to the same extent as the hard-pure disciplines. The use of software during teaching is not very high, however most of the courses have software associated with it particularly in the applied disciplines (both hard and soft), with the soft-applied disciplines more likely to use software in the classroom.

The USA seems more inclined to teach more theoretical aspects of LP such as the revised simplex method and the interior point method whilst New Zealand seems keen to look at the applied aspects, particularly sensitivity analysis using the computer output/ printout.

Further work into this area would look more in depth into how disciplinary differences may influence the learning of students with different types of software. This will present some challenges especially where students' disciplines may not be clear such as in joint honour programmes.

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